Impact of Packetization

on Network Calculus Analysis

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Example Context: TSN

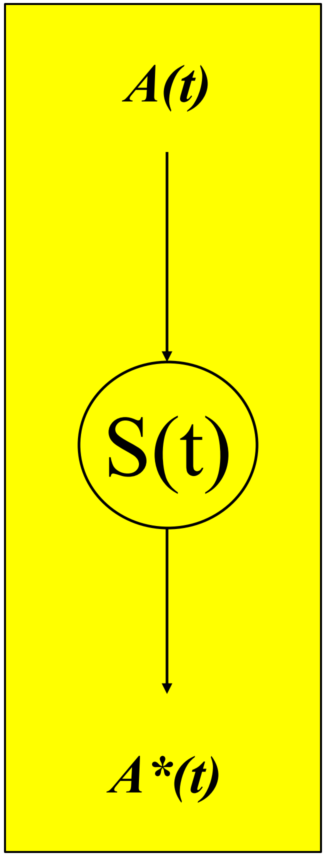
- "7 hops of 100 Mb/s Ethernet if the maximum frame size on the LAN is 1522 bytes" [8021Q-2022]
  - Delay requirement for SR Class A: 2 ms
    - About 300μs per switch
  - Delay requirement for SR Class B: 50 ms
    - About 7ms per switch

- One packet transmission time: 120 μs
  - Same order as the required delay for SR Class A
  - The impact of packetization must be taken into consideration.

Outline

- The System Model
- Impact of packetization in DNC
- Impact of packetization in SNC

The System Model

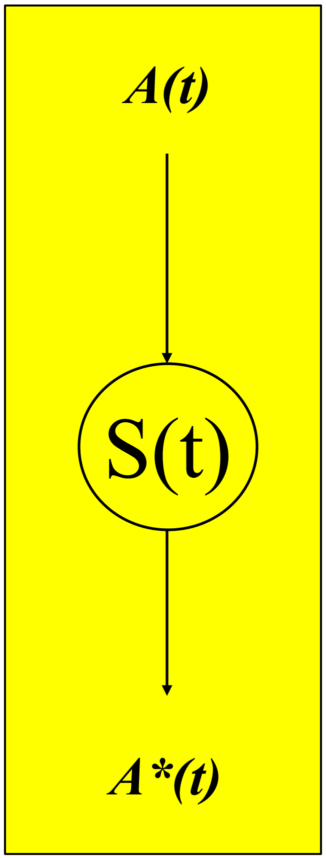


- A packet-switched FIFO system, assuming no loss.
- A packet is said to have arrived to (/served by) the system, when and only when its last bit has arrived to (/ left) the system.
- Input process A(t):
  - *N(t) – number of packets till t*

$$A(t) = \sum_{k=1}^{N(t)} l^k$$

- Service process S(t) and output process A*(t): Similarly defined.

Relation between Output, Input, and Service



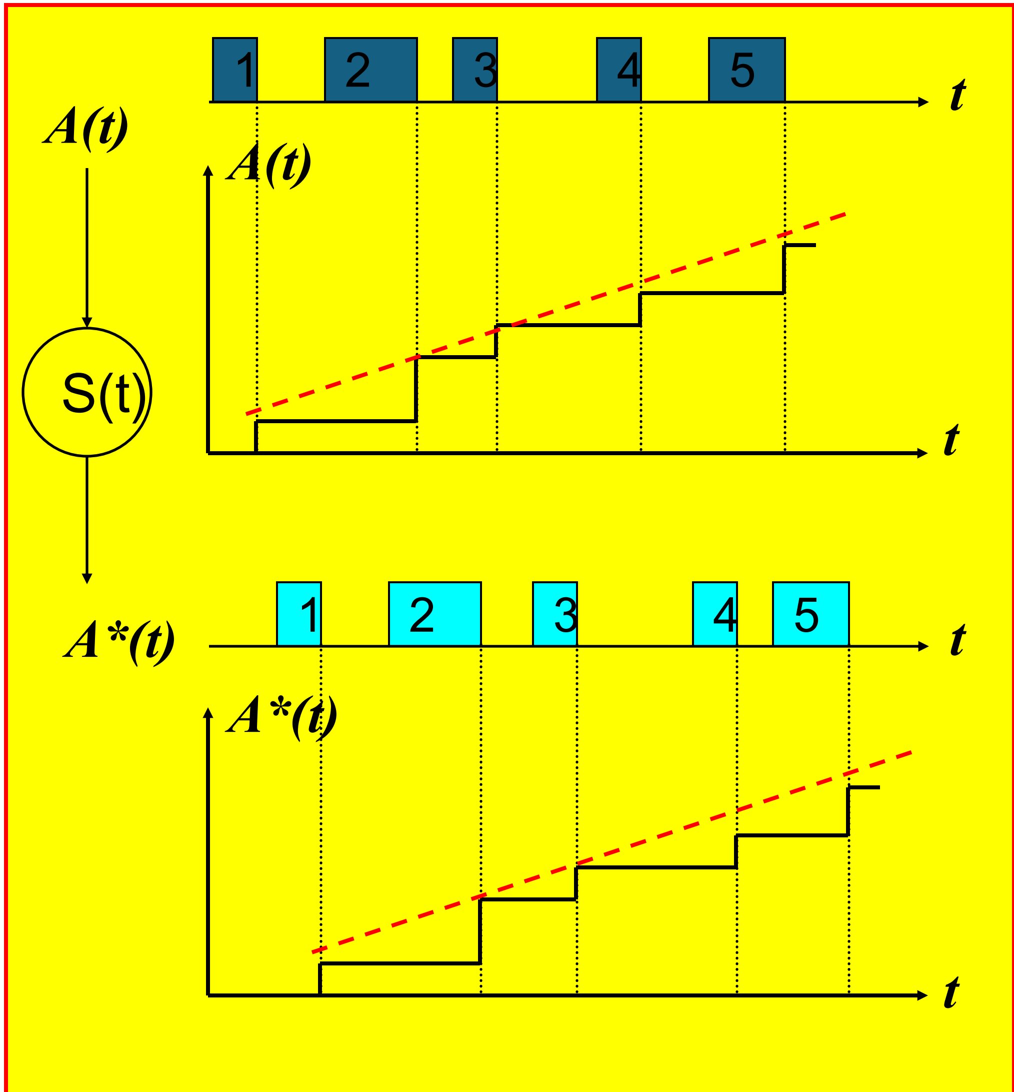
- The Min-Plus Convolution Queueing Principle
  - A fundation for min-plus network calculus

$$A^*(t) = A \otimes S(t)$$

Packetization Effect: Input

- Input process: Stair-case curve
- Impact on arrival curve characterization:
  - The burstiness parameter value σ can never be smaller than maximum packet length.

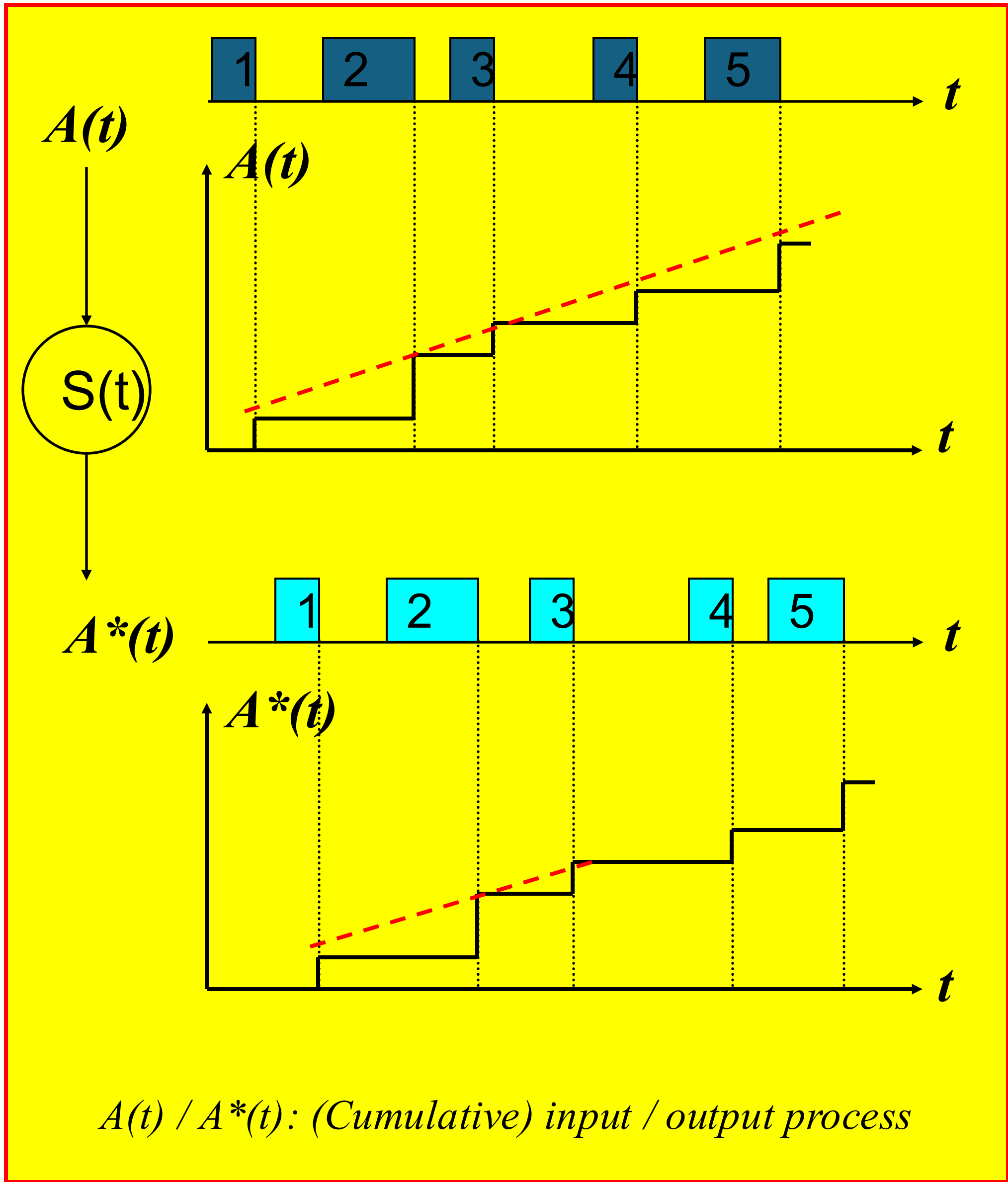
$$\alpha(t) = \begin{cases} \rho t + \sigma, (t > 0) \\ 0, (t = 0) \end{cases}$$



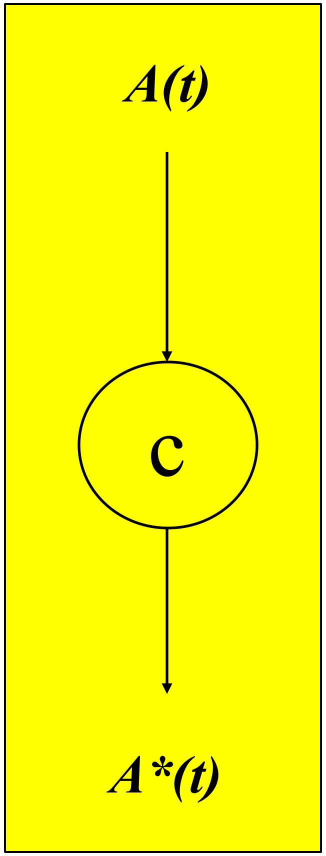
A(t) / A*(t): (Cumulative) input / output process

Packetization Effect: Service

- Capacity v.s. service process
  - Capacity is independent of arrival
  - The service process, how, depends on arrival, e.g. no arrival will lead to no service.
- The process of received service: The output process having a stair-case curve
- Impact on service curve characterization:
  - Care is needed.



The Simplest Case: A link with capacity c (in bps)



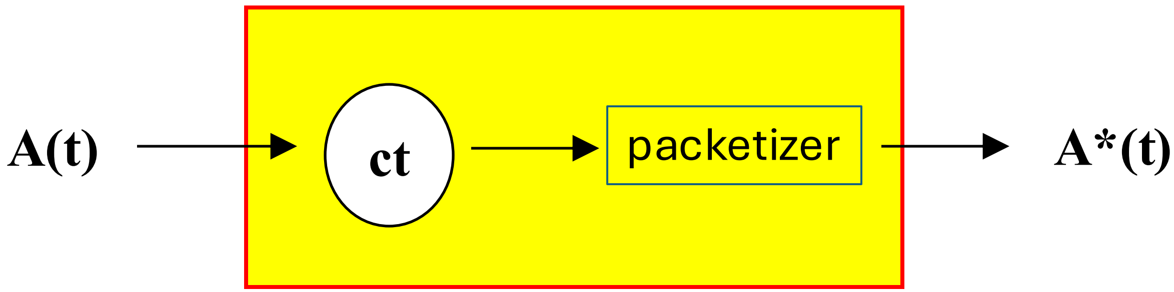
- It has been believed to have the service curve β(t) = ct :

$$\beta(t) = ct$$

$$A^*(t) = A \otimes \beta(t)$$

Valid under fluid traffic or C in packets/sec; not for general packet-switched systems.

Packetizer for DNC Analysis [1]

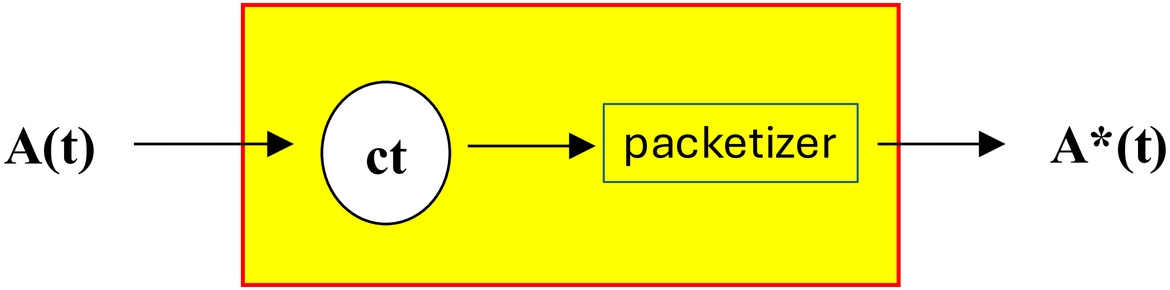


- Corrected service curve $\beta(t) = c\left(t - \frac{l^{max}}{c}\right)^{+}$

- It is indeed also a strict service curve.

[1] C.-S. Chang. *Performance Guarantees in Communication Networks.* Springer-Verlag, 2000.

Impact on Delay Bound Analysis



- Let α(t)=ρt+σ be an arrival curve of A(t) with ρ<c.
- The delay bound from α(t) and $\beta(t)$ is: $\frac{\sigma}{c} + \frac{l^{max}}{c}$
- Delay in the packetizer is bounded in $[\frac{l^{min}}{c}, \frac{l^{max}}{c}]$
- Strictly speaking, ignoring the packetizer only gives improved delay bound: $\frac{\sigma}{c} + \frac{l^{max}}{c} - \frac{l^{min}}{c}$

Packetization Effect: Service

- The difference sastifies, with β(t) = ct, [2]:

$$A \otimes \beta(t) - A^*(t) \leq l^{i(t)}$$

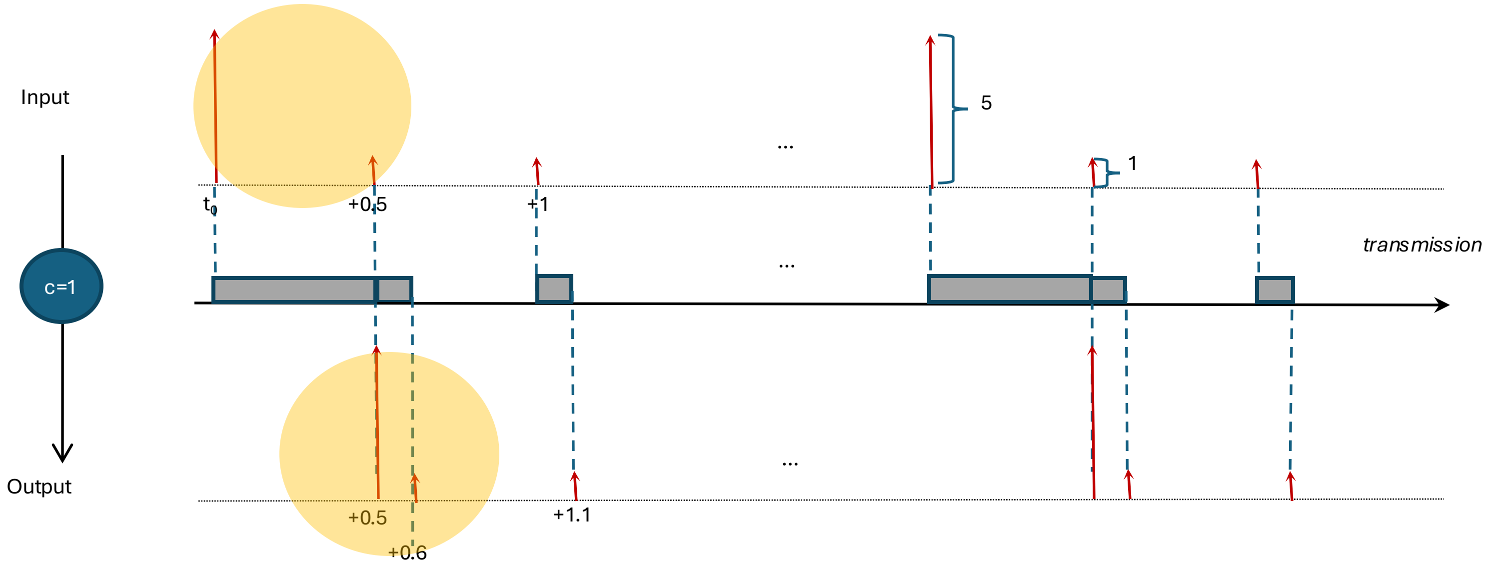
Here, $i(t) = \min\{k : d^k \geq t\}$, which is crucial;

An implication, e.g. [2], is that the delay is bounded by $\frac{\sigma}{c}$

which is better than $\frac{\sigma}{c} + \frac{l^{max}}{c}$ - $\frac{l^{min}}{c}$

[2] Y. Jiang. *Stochastic Service Curve and Delay Bound Analysis - A Single Node Case*. ITC 2013

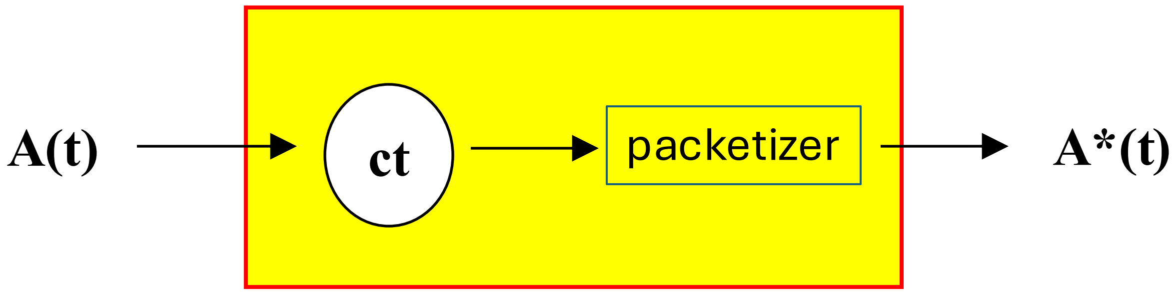
Packetization Effect: Output



- Input: periodic arrivals but different packet lengths
- Service rate: constant c
- ***If ct were service curve, the output would still be periodic.***
- Impact on output burstiness:
  - The burstiness is increased due to packetization.
  - When the corrected service curve $\beta(t) = c\left(t - \frac{l^{max}}{c}\right)^+$ is applied, σ is increased by one maximum packet length [1].

$$\alpha^*(t) = \begin{cases} \rho t + \sigma + lmax, (t > 0) \\ 0, (t = 0) \end{cases}$$

The Packetizer Model



- Can the packetizer approach be applied for output characterization analysis? How?
  - Service curve concatenation

Packetization Effect: DNC Duality

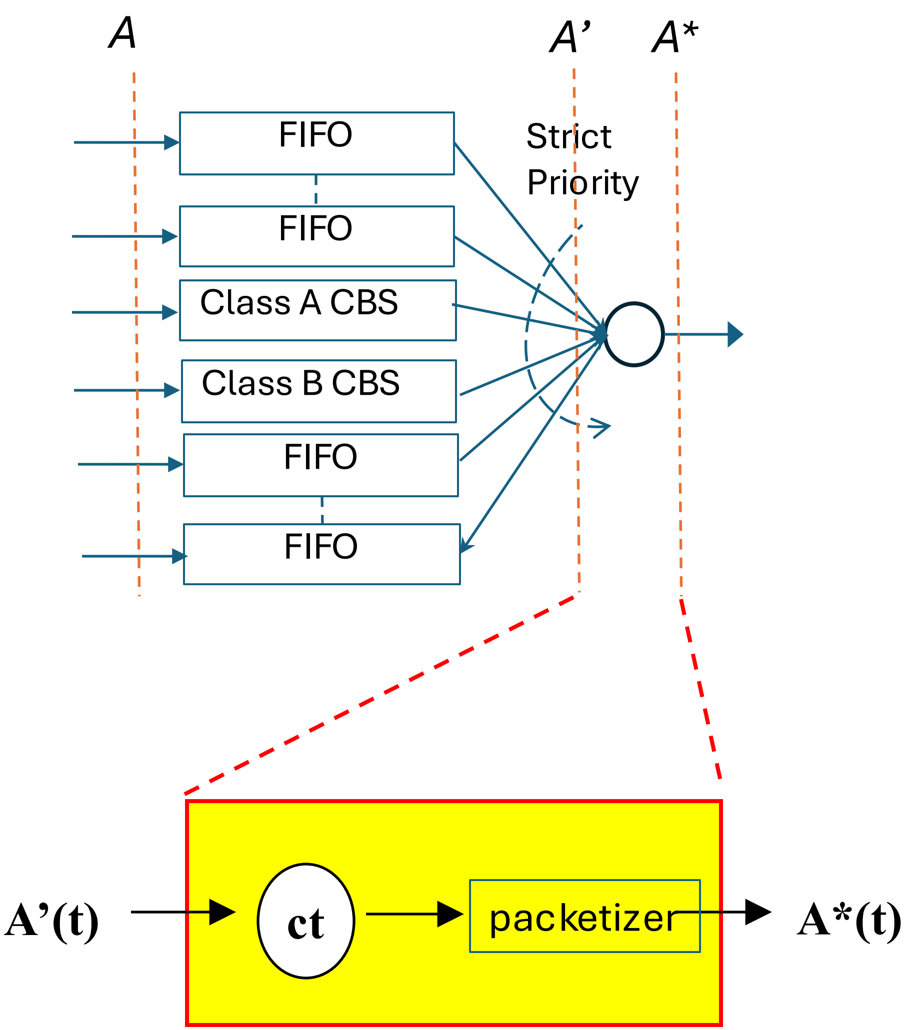
- Duality: $y=f(x) <-> x=f^{-1}(y)$

- Care is needed when applying the duality between min-plus and max-plus DNC [3].

- There exist mappings between min-plus and max-plus arrival curve models [1, 4], so are between min-plus and max-plus service curve models [4].
  - These mappings do not satisfy the duality above, when packetization effect is taken into account.

[3] J. Liebeherr. *Duality of the max-plus and min-plus network calculus.* Foundations and Trends in Networking, 11(3-4):139–282, 2017.
[4] Y. Jiang. *Network calculus bounds for time-sensitive networks: A revisit*. arXiv: 2403.13656, 2024.

Packetization Effect: TSN



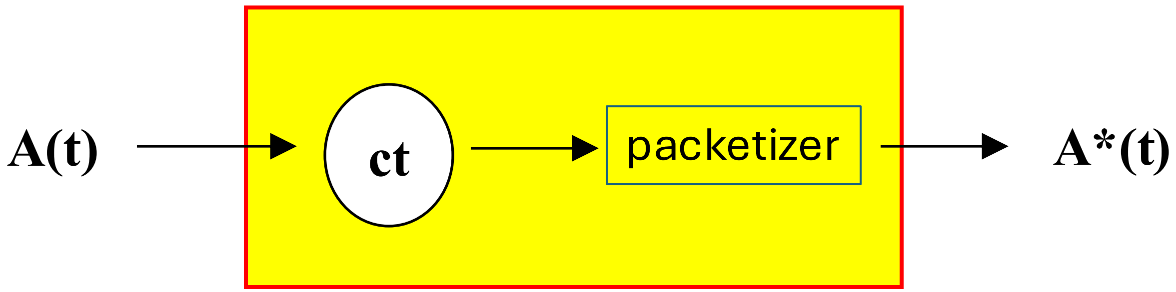
- Is the DNC packetizer approach still applicable?

Care is needed:

- In CBS, packet length affects the schedule: For single CBS on a link with capacity c, *idleSlope x t* is not a service curve [4].

- When SP is used, for the highest priority class, neither *ct* nor $c\left(t - \frac{l_{low\,pri}^{max}}{c}\right)^{+}$ is a service curve [4].

Impact on SNC Analysis



- The simplest case: a link with capacity c (in bps)

- Let $$\beta(t) = ct$$

Packetization Effect: Implicit Dependence between A(t) and S(t)

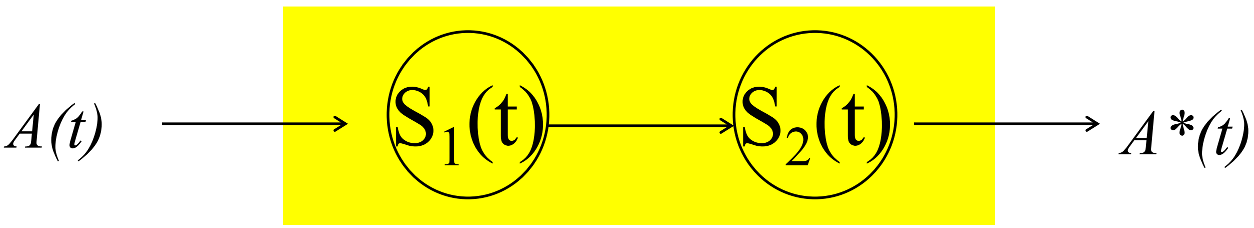
- The difference sastifies, with $\beta(t) = ct$, [2]:

$$A \otimes \beta(t) - A^*(t) \le l^{i(t)}$$

Here, $i(t) = \min\{k : d^k \ge t\}$, which is crucial;

which also imply dependence btw above and $A(t) = \sum_{k=1}^{N(t)} l^k(\mathrm{t})$

Packetization Effect: Tandem System



- A(t), S1(t) and S2(t) are inherently dependent, e.g. Transmission time at different nodes all depends on the same packet length!

$$A(t) = \sum_{k=1}^{N(t)} l^k \qquad S_1(t) = \sum_{k=1}^{N_1(t)} l^k \qquad S_2(t) = \sum_{k=1}^{N_2(t)} l^k$$

- The idea of stochatic strict server and impairement process [5] may be exploited, e.g. extended to / integrated with max-plus domain [2], to enable exploitation of indepence between the two servers.

[5] Y. Jiang, and Y. Liu. *Stochastic Network Calculus*. Springer, 2008

Summary

- Packetization effect can be significant, particularly for some TSN traffic classes

- Impact of packetization in DNC
  - Arrival curve
  - Service curve
  - Delay bound analysis - Ingnoring the effect of last packetizer
  - Output burstiness
  - Duality between min-plus and max-plus DNC
  - TSN schedulers

- Impact of packetization in SNC
  - Single-node
  - Tandem

Impact of Packetization on NC Analysis: A Reminder

Questions?