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Per-Flow Backlog Bounds for a FIFO Server Beyond Token-Bucket-Constrained Arrivals

Lukas Wildberger, Anja Hamscher, Jens B. Schmitt

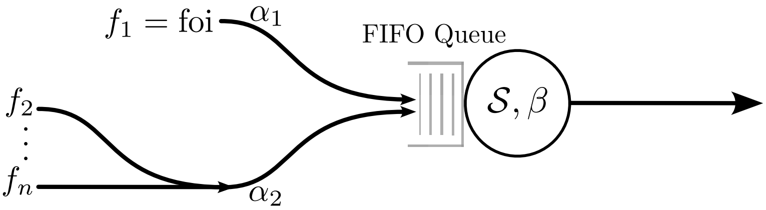
June 5th 2025

These results will be published in QEST + FORMATS 2025

Outline

Motivation

What We're Up To



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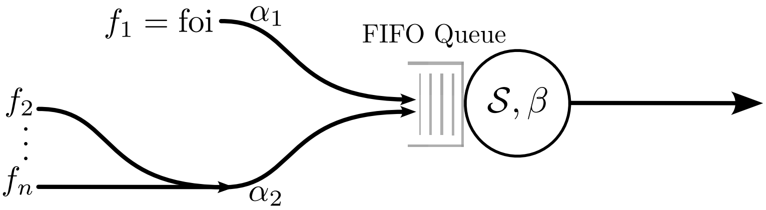

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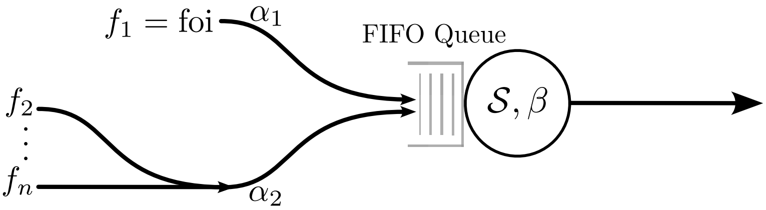

How to choose the free parameter θ in order to minimize the

- Backlog bound
- We have to solve the mathematical problem given by: $\theta_{\text{opt}} = \arg\min_{\theta \geq 0} \{ v(\alpha_1, \beta_\theta^1) \}$

Function Set

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- Literature results primarily for token-bucket arrival and rate-latency service curves [Le Boudec and Thiran, 2001]

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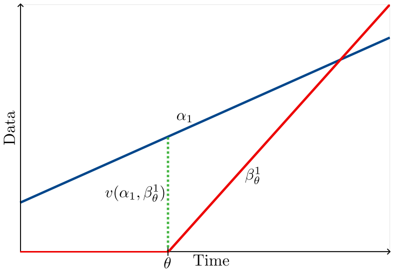

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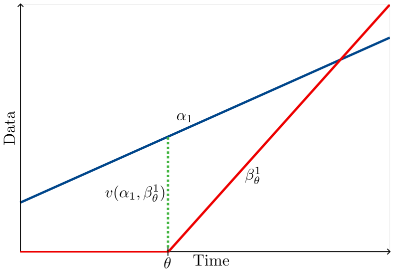
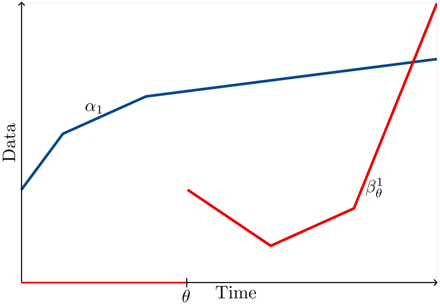
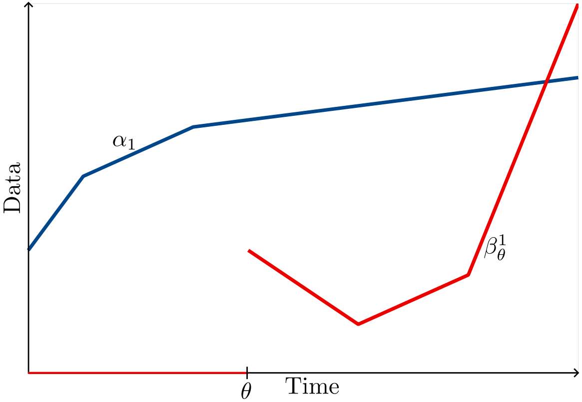




Function Set - PWL Concave/Convex Curves



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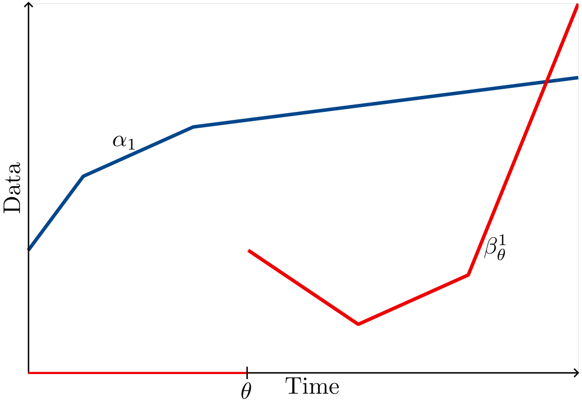
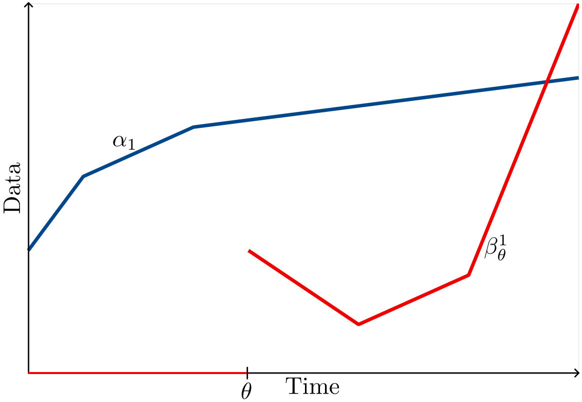


- FIFO residual service curve is also a service curve for the foi **even if it is not wide-sense increasing** [Bouillard et al., 2018]

Terminology



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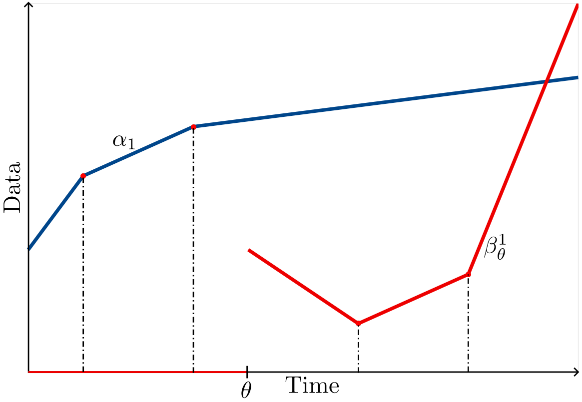

- We call time where the slope of the linear segment changes "breakpoint"
  - In the literature also referred to as "intersetion point" or "changepoint"

Backlog Bound

Backlog Bound: Non-Wide-Sense Increasing Property

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Let $\alpha, \beta \in \mathcal{F}$ be given. Then it holds that $v(\alpha, \beta) = v(\alpha, \beta_\downarrow)$, with $\beta_\downarrow := \beta \overline{\oslash} 0$.

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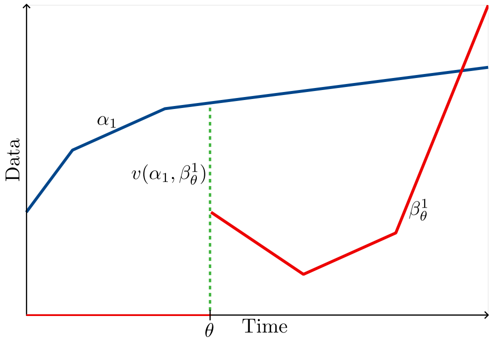

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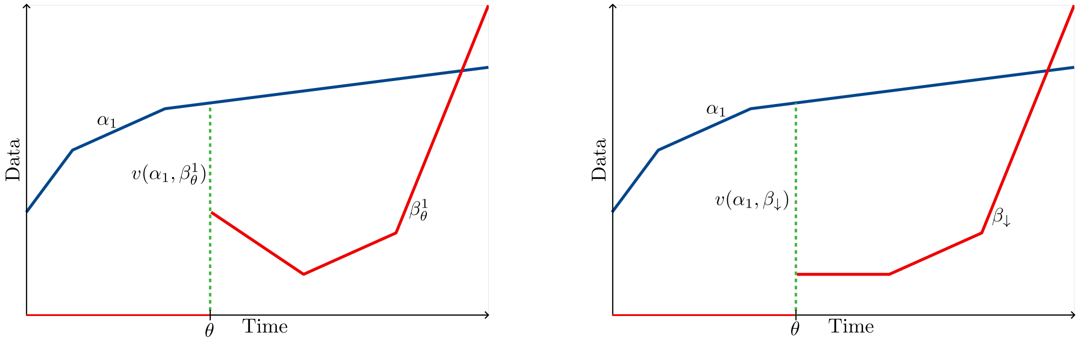

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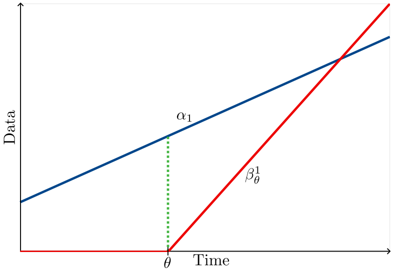

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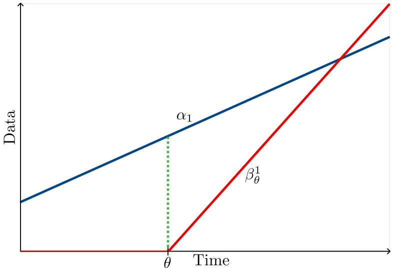

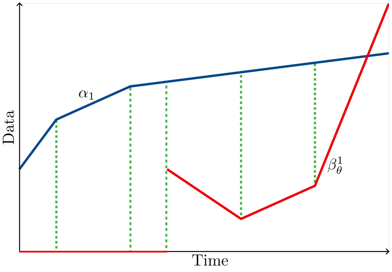

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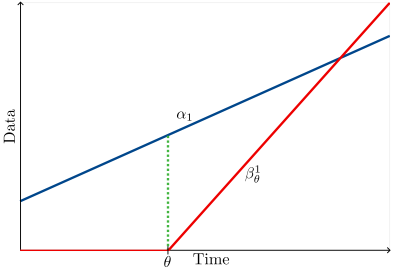

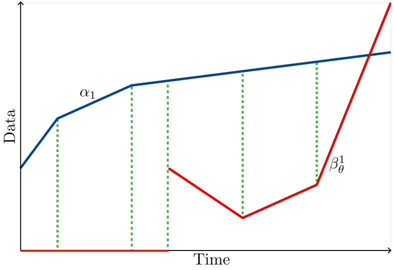

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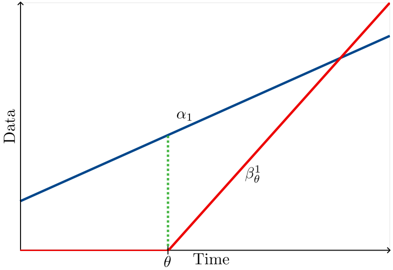

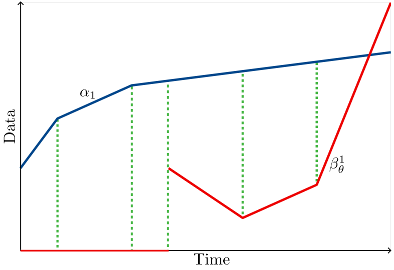

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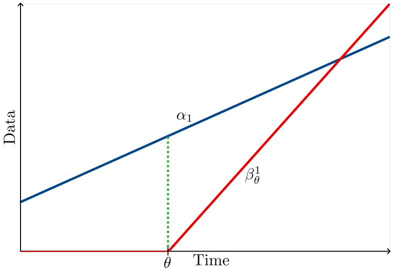

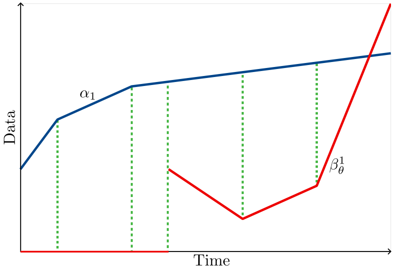

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- We have to take care of all linear sections at once

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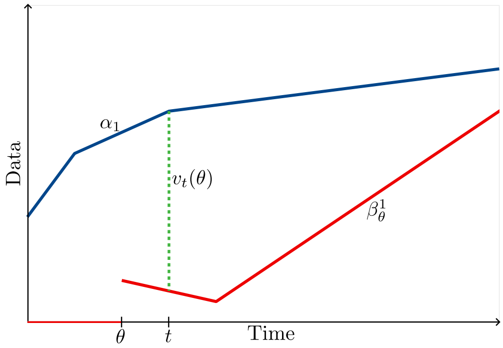

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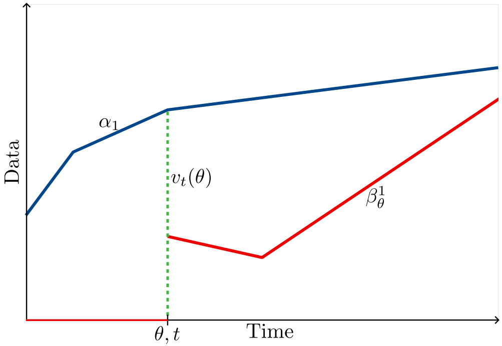

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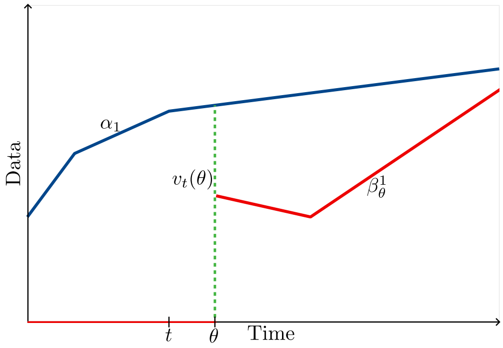

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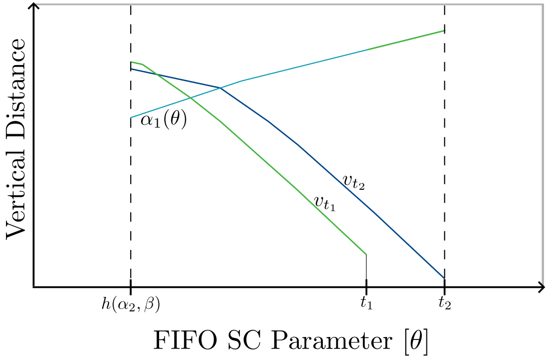

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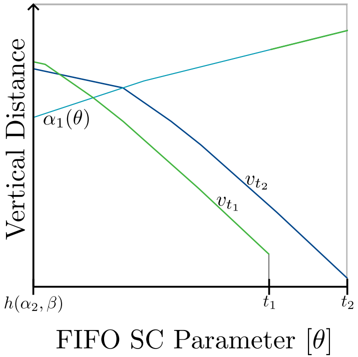

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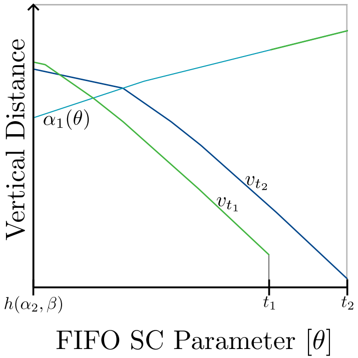

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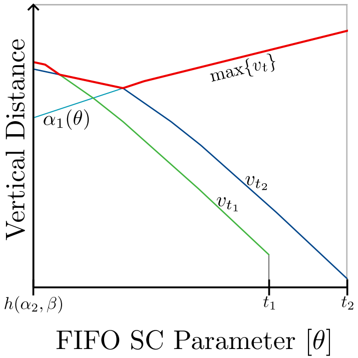

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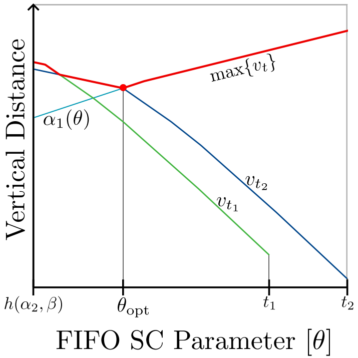

Numerical Evaluation

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- Setting:
  - Rate-latency service curve $\beta_{R,T}$
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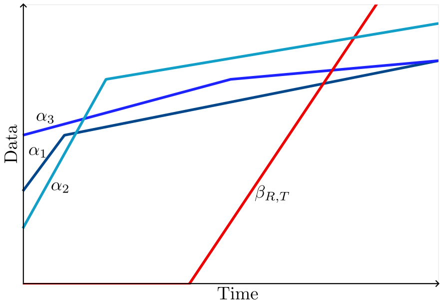

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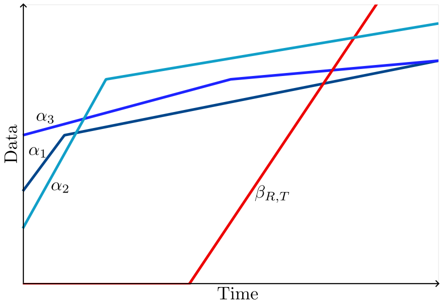

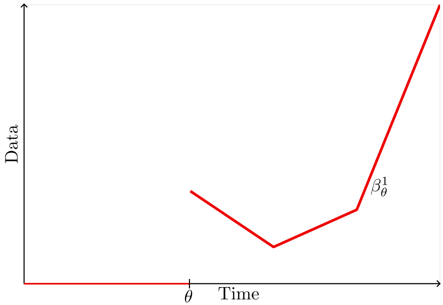

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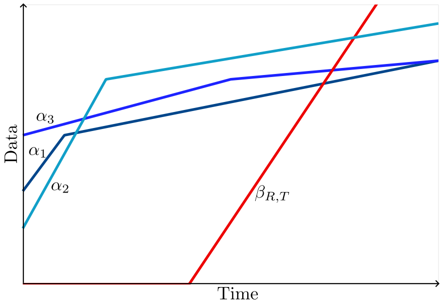

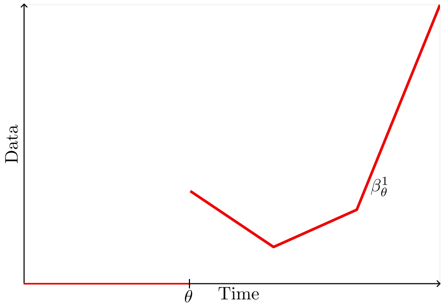

- The parameters used, b_i^1, r_i^1, b_i^2 and r_i^2, are randomly chosen out of a predefined interval

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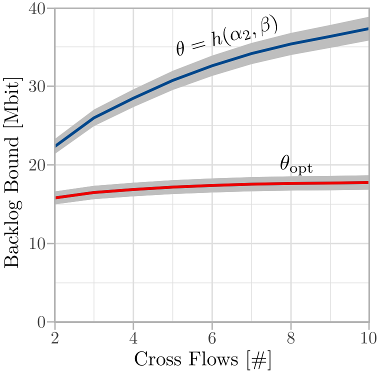

Conclusion

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What's next?

- Explore (further) applications
- Consider multi-node case
- ...

References

Bouillard, A., Boyer, M., and Le Corronc, E. (2018).
Deterministic Network Calculus: From Theory to Practical Implementation.
John Wiley & Sons.

Le Boudec, J.-Y. and Thiran, P. (2001).
Network Calculus: A Theory of Deterministic Queuing Systems for the Internet.
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Delay-Optimal θ (Theoretical Result)

Theorem

Let $\mathcal{S}$ be a system that multiplexes two flows f_1 and f_2 according to FIFO, where the arrivals of f_1 and f_2, A_1 and A_2, are constrained by α_1 and α_2 respectively. Further, assume that $\mathcal{S}$ offers a service curve β to the aggregate of the flows. If $\alpha_1 \in \mathcal{F}$ and $\alpha_2 \in \mathcal{F}$ are concave and $\beta \in \mathcal{F}$ is convex, then the optimal θ, that minimizes the delay bound, is given by

$$\theta_{opt} = \arg\min_{\theta \geq 0}\{h(\alpha_1, \beta_\theta^1)\} = h(\alpha_{\mathrm{agg}}, \beta).$$

- Under FIFO scheduling the delay bound of the aggregate equals the per-flow delay bound
- It can be directly shown that $h(\alpha_{\mathrm{agg}}, \beta) = h\left(\alpha_1, \beta^1_{h(\alpha_{\mathrm{agg}}, \beta)}\right)$

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$$= \begin{cases} \alpha_1(t) - \beta(t) + \alpha_2(t-\theta), & \text{if } h(\alpha_2,\beta) \le \theta < t, \\ \alpha_1(\theta), & \text{if } \theta \ge t > h(\alpha_2,\beta). \end{cases}$$

By definition we have $\beta_\theta^1 = [\beta(t) - \alpha_2(t-\theta)]^+ \wedge \delta_\theta(t)$. Since for $\theta < t$ it holds that $\delta_\theta(t) = 0$, and because $h(\alpha_2,\beta) \le \theta$ ensures that $\beta(t) \ge \alpha_2(t-\theta)$ for all t, both the positive part and $\delta_\theta(t)$ can be omitted.

Application

We are now able to properly choose flow queue sizes for a setting with flows, having their own flow queue, getting FIFO scheduled